

## Event Hopping (events)

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**Subtask 1.** For every event, you can switch to at most one other event.

Consider a directed graph where events correspond to nodes and there is an edge from event  $i$  to event  $j$  iff you can switch from  $i$  to  $j$ . This graph can be efficiently constructed in  $\mathcal{O}(N \log N)$  with a sweep line since there are at most  $N$  edges.

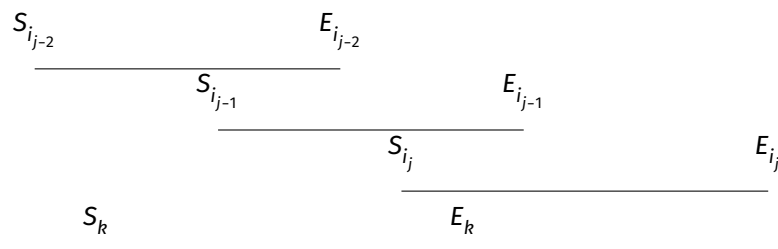
For simplicity, assume that there is only one component. Note that there is a case that prevents the graph from being a directed tree. If there are two events  $a$  and  $b$  with the same endpoint, i.e.  $E_a = E_b$  and  $S_a \leq S_b$ , we have to handle queries that end at the shorter event  $b$  separately. In any optimal sequence of events, we first visit event  $a$  and then event  $b$ . Therefore, we can solve this special case by replacing queries of the form  $(-, b)$  with  $(-, a)$ . This allows us to reduce the graph to a directed tree.

To answer a query  $(s, e)$ , we need to check that  $e$  is a parent of  $s$  and then compute the minimum number of event switches using the depth of  $e$  and  $s$ . We can precompute the depth of each node in linear time with a DFS and with an inorder traversal of the tree we can answer in constant time whether  $e$  is a parent of  $s$  or not. Therefore, the whole precomputation can be done in linear time and we can solve this subtask in  $\mathcal{O}(N \log N + Q)$ .

**Subtask 2.**  $N \leq 1\,000$  and  $Q \leq 100$

This subtask can be solved by brute force. If we do a naive BFS for each query, we get a solution with a runtime of  $\mathcal{O}(QN^2)$ .

For the following subtasks we need an important observation. Assume that we attend an event  $s$  and want to attend an event  $e$ . Let  $s = i_1, i_2, \dots, i_l = e$  be a sequence of events with a minimum number of event switches. We can prove that if  $j > 2$ , we can always replace in this sequence event  $i_{j-1}$  with event  $k$  such that it is possible to switch from  $k$  to  $i_j$  and  $S_k$  is the smallest possible. Note that  $E_{i_{j-2}} < S_{i_j}$  (otherwise we could switch directly from  $i_{j-2}$  to  $i_j$ ) and  $S_{i_{j-1}} \leq E_{i_{j-2}}$  (since we can switch from  $i_{j-2}$  to  $i_{j-1}$ ). Combining it with the fact that  $S_k \leq S_{i_{j-1}}$  and  $E_k \geq S_{i_j}$ , we can conclude that it is possible to switch from  $i_{j-2}$  to  $k$ .



**Subtask 3.**  $N \leq 5\,000$

This subtask can be solved by precomputing all possible answers. How do we do this faster than  $\mathcal{O}(N^3)$ ?



Let us assume that all events are sorted in non-decreasing order by their end time and let  $\text{switches}(j, i)$  be the minimum number of event switches if we attend event  $j$  and want to attend event  $i$ . We now compute all answers with a fixed ending event  $i$  by sweeping over the events in decreasing order of their index. Furthermore, we use a set where we store the pairs  $(\text{switches}(j, i), j)$  for all events  $j$  to which we can currently switch. If we want to find  $\text{switches}(k, i)$ , we have to find an event  $j$  with minimal  $\text{switches}(j, i)$ , so that we can switch from  $k$  to  $j$ . We can quickly find this event using our set. Thus, it is possible to precompute all answers in  $\mathcal{O}(N^2 \log N)$  and answer queries in constant time.

It is also possible to avoid a set and improve the runtime to  $\mathcal{O}(N^2)$  by noticing that  $\text{switches}(-, i)$  is a monotone function.

Alternatively, we can use some slow precomputation to find the event  $k$  from the observation above and answer queries using binary jumping.

#### **Subtask 4.** $Q \leq 100$

This subtask can be solved by precomputing for each event  $i$  the event  $k$  such that it is possible to switch from  $k$  to  $i$  and  $S_k$  is the smallest possible. To find event  $k$  fast, we can use a segment tree/sparse table that allows us to query the minimum start time of all events that end in  $[S_i, E_i]$ . Alternatively, we can iterate over all events in non-decreasing order of their end time and use a monotonic stack to find event  $k$  with a binary search.

We can now answer each query  $(s, e)$  in linear time by starting at event  $e$  and „switching backward“ with the precomputed information until we reach event  $s$ . This leads to a solution with complexity  $\mathcal{O}(QN)$ .

#### **Subtask 5.** No event is completely contained in another event, i.e. there are no two events $i \neq j$ with $S_i \leq S_j < E_j \leq E_i$ .

Assume that we want to answer a query  $(s, e)$ . It is always optimal to switch to an event  $k$  with maximum  $E_k$ . The constraints in this subtask ensure that we can apply the same argument as in the above observation. Therefore, we can find with a sweep line for each event  $i$  the event  $k$  such that we can switch from  $i$  to  $k$  and  $E_k$  is the largest possible. We can now compute an array  $\text{next}[m][i]$  that stores the event that we reach if we start at event  $i$  and perform  $2^m$  event switches. This allows us to answer queries in  $\mathcal{O}(\log N)$  using binary jumping.

#### **Subtask 6.** No further constraints.

To solve this subtask, we combine the solutions of the previous two subtasks. That means we first precompute the event  $k$  such that  $S_k$  is the smallest possible and we can switch from event  $k$  to event  $i$  using a sweep line and a segment tree/sparse table. Then we compute an array  $\text{prev}[m][i]$  that stores the event that we reach if we start at event  $i$  and perform  $2^m$  ‘backward event switches.’ The final complexity is  $\mathcal{O}((N + Q) \log N)$ .